XXI. *Experiments to determine the Density of the Earth. By* Henry Cavendish, *Esq. F. R. S. and A. S.*

Read June 21, 1798.

Many years ago, the late Rev. John Michell, of this Society, contrived a method of determining the density of the earth, by rendering sensible the attraction of small quantities of matter; but, as he was engaged in other pursuits, he did not complete the apparatus till a short time before his death, and did not live to make any experiments with it. After his death, the apparatus came to the Rev. Francis John Hyde Wollaston, Jacksonian Professor at Cambridge, who, not having conveniences for making experiments with it, in the manner he could wish, was so good as to give it to me.

The apparatus is very simple; it consists of a wooden arm, 6 feet long, made so as to unite great strength with little weight. This arm is suspended in an horizontal position, by a slender wire 40 inches long, and to each extremity is hung a leaden ball, about 2 inches in diameter; and the whole is inclosed in a narrow wooden case, to defend it from the wind.

As no more force is required to make this arm turn round on its centre, than what is necessary to twist the suspending wire, it is plain, that if the wire is sufficiently slender, the most minute force, such as the attraction of a leaden weight a few inches in diameter, will be sufficient to draw the arm sensibly aside. The weights which Mr. Michell intended to use were

8 inches diameter. One of these was to be placed on one side the case, opposite to one of the balls, and as near it as could conveniently be done, and the other on the other side, opposite to the other ball, so that the attraction of both these weights would conspire in drawing the arm aside; and, when its position, as affected by these weights, was ascertained, the weights were to be removed to the other side of the case, so as to draw the arm the contrary way, and the position of the arm was to be again determined; and, consequently, half the difference of these positions would shew how much the arm was drawn aside by the attraction of the weights.

In order to determine from hence the density of the earth, it is necessary to ascertain what force is required to draw the arm aside through a given space. This Mr. MICHELL intended to do, by putting the arm in motion, and observing the time of its vibrations, from which it may easily be computed.*

Mr. MICHELL had prepared two wooden stands, on which the leaden weights were to be supported, and pushed forwards, till they came almost in contact with the case; but he seems to have intended to move them by hand.

As the force with which the balls are attracted by these weights is excessively minute, not more than $\frac{1}{50,000,000}$ of their weight, it is plain, that a very minute disturbing force will be sufficient to destroy the success of the experiment; and, from the following experiments it will appear, that the disturbing

---

* Mr. COULOMB has, in a variety of cases, used a contrivance of this kind for trying small attractions; but Mr. MICHELL informed me of his intention of making this experiment, and of the method he intended to use, before the publication of any of Mr. COULOMB's experiments.

force most difficult to guard against, is that arising from the variations of heat and cold; for, if one side of the case is warmer than the other, the air in contact with it will be rarefied, and, in consequence, will ascend, while that on the other side will descend, and produce a current which will draw the arm sensibly aside.[*]

As I was convinced of the necessity of guarding against this source of error, I resolved to place the apparatus in a room which should remain constantly shut, and to observe the motion of the arm from without, by means of a telescope; and to suspend the leaden weights in such manner, that I could move them without entering into the room. This difference in the manner of observing, rendered it necessary to make some alteration in Mr. MICHELL's apparatus; and, as there were some parts of it which I thought not so convenient as could be wished, I chose to make the greatest part of it afresh.

Fig. 1. (Tab. XXIII.) is a longitudinal vertical section through the instrument, and the building in which it is placed. ABCDDCBAEFFE, is the case; x and x are the two balls, which are suspended by the wires bx from the arm $ghmh$, which is itself suspended by the slender wire gl. This arm consists of a slender deal rod hmh, strengthened by a silver

---

[*] M. CASSINI, in observing the variation compass placed by him in the Observatory, (which was constructed so as to make very minute changes of position visible, and in which the needle was suspended by a silk thread,) found that standing near the box, in order to observe, drew the needle sensibly aside; which I have no doubt was caused by this current of air. It must be observed, that his compass-box was of metal, which transmits heat faster than wood, and also was many inches deep; both which causes served to increase the current of air. To diminish the effect of this current, it is by all means advisable to make the box, in which the needle plays, not much deeper than is necessary to prevent the needle from striking against the top and bottom.

wire *hgb*; by which means it is made strong enough to support the balls, though very light.*

The case is supported, and set horizontal, by four screws, resting on posts fixed firmly into the ground: two of them are represented in the figure, by S and S; the two others are not represented, to avoid confusion. GG and GG are the end walls of the building. W and W are the leaden weights; which are suspended by the copper rods R*r*P*r*R, and the wooden bar *rr*, from the centre pin P*p*. This pin passes through a hole in the beam HH, perpendicularly over the centre of the instrument, and turns round in it, being prevented from falling by the plate *p*. MM is a pulley, fastened to this pin; and M*m*, a cord wound round the pulley, and passing through the end wall; by which the observer may turn it round, and thereby move the weights from one situation to the other.

Fig. 2. (Tab. XXIV.) is a plan of the instrument. AAAA is the case. SSSS, the four screws for supporting it. *bb*, the arm and balls. W and W, the weights. MM, the pulley for moving them. When the weights are in this position, both conspire in drawing the arm in the direction *b*W; but, when they are removed to the situation *w* and *w*, represented by the dotted lines, both conspire in drawing the arm in the contrary direction *bw*. These weights are prevented from striking the instrument, by pieces of wood, which stop them as soon as they come within $\frac{1}{5}$ of an inch of the

---

* Mr. MICHELL's rod was entirely of wood, and was much stronger and stiffer than this, though not much heavier; but, as it had warped when it came to me, I chose to make another, and preferred this form, partly as being easier to construct and meeting with less resistance from the air, and partly because, from its being of a less complicated form, I could more easily compute how much it was attracted by the weights.

case. The pieces of wood are fastened to the wall of the build-
ing; and I find, that the weights may strike against them with
considerable force, without sensibly shaking the instrument.

In order to determine the situation of the arm, slips of ivory
are placed within the case, as near to each end of the arm as
can be done without danger of touching it, and are divided to
20ths of an inch. Another small slip of ivory is placed at each
end of the arm, serving as a vernier, and subdividing these
divisions into 5 parts; so that the position of the arm may be
observed with ease to 100ths of an inch, and may be estimated
to less. These divisions are viewed, by means of the short tele-
scopes T and T, (fig. 1.) through slits cut in the end of the case,
and stopped with glass; they are enlightened by the lamps L
and L, with convex glasses, placed so as to throw the light on
the divisions; no other light being admitted into the room.

The divisions on the slips of ivory run in the direction W*w*,
(fig. 2.) so that, when the weights are placed in the positions
w and *w*, represented by the dotted circles, the arm is drawn
aside, in such direction as to make the index point to a higher
number on the slips of ivory; for which reason, I call this the
positive position of the weights.

FK, (fig. 1.) is a wooden rod, which, by means of an endless
screw, turns round the support to which the wire *gl* is fastened,
and thereby enables the observer to turn round the wire, till the
arm settles in the middle of the case, without danger of touch-
ing either side. The wire *gl* is fastened to its support at top,
and to the centre of the arm at bottom, by brass clips, in which
it is pinched by screws.

In these two figures, the different parts are drawn nearly in

MDCCXCVIII.   3 P

the proper proportion to each other, and on a scale of one to thirteen.

Before I proceed to the account of the experiments, it will be proper to say something of the manner of observing. Suppose the arm to be at rest, and its position to be observed, let the weights be then moved, the arm will not only be drawn aside thereby, but it will be made to vibrate, and its vibrations will continue a great while; so that, in order to determine how much the arm is drawn aside, it is necessary to observe the extreme points of the vibrations, and from thence to determine the point which it would rest at if its motion was destroyed, or the point of rest, as I shall call it. To do this, I observe three successive extreme points of a vibration, and take the mean between the first and third of these points, as the extreme point of vibration in one direction, and then assume the mean between this and the second extreme, as the point of rest; for, as the vibrations are continually diminishing, it is evident, that the mean between two extreme points will not give the true point of rest.

It may be thought more exact, to observe many extreme points of vibration, so as to find the point of rest by different sets of three extremes, and to take the mean result; but it must be observed, that notwithstanding the pains taken to prevent any disturbing force, the arm will seldom remain perfectly at rest for an hour together; for which reason, it is best to determine the point of rest, from observations made as soon after the motion of the weights as possible.

The next thing to be determined is the time of vibration, which I find in this manner: I observe the two extreme points of a vibration, and also the times at which the arm arrives at

two given divisions between these extremes, taking care, as well as I can guess, that these divisions shall be on different sides of the middle point, and not very far from it. I then compute the middle point of the vibration, and, by proportion, find the time at which the arm comes to this middle point. I then, after a number of vibrations, repeat this operation, and divide the interval of time, between the coming of the arm to these two middle points, by the number of vibrations, which gives the time of one vibration. The following example will explain what is here said more clearly.

Extreme Points.	Division.	Time.			Point of rest.	Time of middle of vibration.
27,2		h.	′	″		h. ′ ″
	25	10	23	4 ⎱	-	10 23 23
	24			57 ⎰		
22,1	-	-	-	-	24,6	
27	-	-	-	-	24,7	
22,6	-	-	-	-	24,75	
26,8	-	-	-	-	24,8	
23	-	-	-	-	24,85	
26,6	-	-	-	-	24,9	
	25	11	5	22 ⎱	-	11 5 22
	24		6	48 ⎰		
23,4						

The first column contains the extreme points of the vibrations. The second, the intermediate divisions. The third, the time at which the arm came to these divisions; and the fourth, the point of rest, which is thus found: the mean between the first and third extreme points is 27,1, and the mean between this and the second extreme point is 24,6, which is the point of rest, as found by the three first extremes. In like manner, the

point of rest found by the second, third, and 4th extremes, is 24,7, and so on. The fifth column is the time at which the arm came to the middle point of the vibration, which is thus found: the mean between 27,2 and 22,1 is 24,65, and is the middle point of the first vibration; and, as the arm came to 25 at 10^h 23' 4", and to 24 at 10^h 23' 57", we find, by proportion, that it came to 24,65 at 10^h 23' 23". In like manner, the arm came to the middle of the seventh vibration at 11^h 5' 22"; and, therefore, six vibrations were performed in 41' 59", or one vibration in 7' 0".

To judge of the propriety of this method, we must consider in what manner the vibration is affected by the resistance of the air, and by the motion of the point of rest.

Let the arm, during the first vibration, move from D to B, (Tab. XXIV. fig. 3.) and, during the second, from B to d; Bd being less than DB, on account of the resistance. Bisect DB in M, and Bd in m, and bisect Mm in n, and let x be any point in the vibration; then, if the resistance is proportional to the square of the velocity, the whole time of a vibration is very little altered; but, if T is taken to the time of one vibration, as the diameter of a circle to its semicircumference, the time of moving from B to n exceeds $\frac{1}{2}$ a vibration, by $\frac{T \times D d}{8 \, B n}$ nearly; and the time of moving from B to m falls short of $\frac{1}{2}$ a vibration, by as much; and the time of moving from B to x, in the second vibration, exceeds that of moving from x to B, in the first, by $\frac{T \times D d \times B x^2}{4 \, B n^2 \times \sqrt{B x} \times x \delta}$, supposing D$d$ to be bisected in δ; so that, if a mean is taken, between the time of the first arrival of the arm at x and its returning back to the same point, this mean will be earlier than the true time of its coming to B, by $\frac{T \times D d \times B x^2}{8 \, B n^2 \sqrt{B x} \times x \delta}$.

The effect of motion in the point of rest is, that when the arm is moving in the same direction as the point of rest, the time of moving from one extreme point of vibration to the other is increased, and it is diminished when they are moving in contrary directions; but, if the point of rest moves uniformly, the time of moving from one extreme to the middle point of the vibration, will be equal to that of moving from the middle point to the other extreme, and moreover, the time of two successive vibrations will be very little altered; and, therefore, the time of moving from the middle point of one vibration to the middle point of the next, will also be very little altered.

It appears, therefore, that on account of the resistance of the air, the time at which the arm comes to the middle point of the vibration, is not exactly the mean between the times of its coming to the extreme points, which causes some inaccuracy in my method of finding the time of a vibration. It must be observed, however, that as the time of coming to the middle point is before the middle of the vibration, both in the first and last vibration, and in general is nearly equally so, the error produced from this cause must be inconsiderable; and, on the whole, I see no method of finding the time of a vibration which is liable to less objection.

The time of a vibration may be determined, either by previous trials, or it may be done at each experiment, by ascertaining the time of the vibrations which the arm is actually put into by the motion of the weights; but there is one advantage in the latter method, namely, that if there should be any accidental attraction, such as electricity, in the glass plates through which the motion of the arm is seen, which should increase the force necessary to draw the arm aside, it would also dimi-

nish the time of vibration; and, consequently, the error in the result would be much less, when the force required to draw the arm aside was deduced from experiments made at the time, than when it was taken from previous experiments.

Account of the Experiments.

In my first experiments, the wire by which the arm was suspended was $39\frac{1}{4}$ inches long, and was of copper silvered, one foot of which weighed $2\frac{4}{10}$ grains : its stiffness was such, as to make the arm perform a vibration in about 15 minutes. I immediately found, indeed, that it was not stiff enough, as the attraction of the weights drew the balls so much aside, as to make them touch the sides of the case; I, however, chose to make some experiments with it, before I changed it.

In this trial, the rods by which the leaden weights were suspended were of iron; for, as I had taken care that there should be nothing magnetical in the arm, it seemed of no signification whether the rods were magnetical or not; but, for greater security, I took off the leaden weights, and tried what effect the rods would have by themselves. Now I find, by computation, that the attraction of gravity of these rods on the balls, is to that of the weights, nearly as 17 to 2500; so that, as the attraction of the weights appeared, by the foregoing trial, to be sufficient to draw the arm aside by about 15 divisions, the attraction of the rods alone should draw it aside about $\frac{1}{10}$ of a division; and, therefore, the motion of the rods from one near position to the other, should move it about $\frac{1}{5}$ of a division.

The result of the experiment was, that for the first 15 minutes after the rods were removed from one near position to the other, very little motion was produced in the arm, and

hardly more than ought to be produced by the action of gravity; but the motion then increased, so that, in about a quarter or half an hour more, it was found to have moved $\frac{1}{2}$ or $1\frac{1}{2}$ division, in the same direction that it ought to have done by the action of gravity. On returning the irons back to their former position, the arm moved backward, in the same manner that it before moved forward.

It must be observed, that the motion of the arm, in these experiments, was hardly more than would sometimes take place without any apparent cause; but yet, as in three experiments which were made with these rods, the motion was constantly of the same kind, though differing in quantity from $\frac{1}{2}$ to $1\frac{1}{2}$ division, there seems great reason to think that it was produced by the rods.

As this effect seemed to me to be owing to magnetism, though it was not such as I should have expected from that cause, I changed the iron rods for copper, and tried them as before; the result was, that there still seemed to be some effect of the same kind, but more irregular, so that I attributed it to some accidental cause, and therefore hung on the leaden weights, and proceeded with the experiments.

It must be observed, that the effect which seemed to be produced by moving the iron rods from one near position to the other, was, at a medium, not more than one division; whereas the effect produced by moving the weight from the midway to the near position, was about 15 divisions; so that, if I had continued to use the iron rods, the error in the result caused thereby, could hardly have exceeded $\frac{1}{30}$ of the whole.

EXPERIMENT I.   Aug. 5.

Weights in midway position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.				Difference.	
		h.	′	″		h.	′	″		′	″
11,4		9	42	0							
11,5			55	0							
11,5		10	5	0		11,5					

At 10^h 5′, weights moved to positive position.

23,4							
27,6		–	–	–	25,82		
24,7		–	–	–	26,07		
27,3		–	–	–	26,1		
25,1		–	–	–			

At 11^h 6′, weights returned back to midway position.

Extreme points	Divisions	Time			Point of rest	Time of mid. of vib.			Difference	
5,										
	11	0	0	48 }	–	0	1	13		
	12		1	30 }						
18,2					12	–	–		14	56
	12		16	29 }	–		16	9		
	11		17	20 }						
6,6		–	–	–	11,92	–	–		14	36
	11		30	24 }	–		30	45		
	12		31	11 }						
16,3		–	–	–	11,72	–	–		15	13
	12		45	58 }	–		45	58		
	11		47	4 }						
7,7										

Motion on moving from midway to pos.   = 14,32

                 pos. to midway   –   –   = 14,1

Time of one vibration   –   –   = 14′ 55″

It must be observed, that in this experiment, the attraction of the weights drew the arm from 11,5 to 25,8, so that, if no contrivance had been used to prevent it, the momentum acquired thereby would have carried it to near 40, and would, therefore, have made the balls to strike against the case. To prevent this, after the arm had moved near 15 divisions, I returned the weights to the midway position, and let them remain there, till the arm came nearly to the extent of its vibration, and then again moved them to the positive position, whereby the vibrations were so much diminished, that the balls did not touch the sides; and it was this which prevented my observing the first extremity of the vibration. A like method was used, when the weights were returned to the midway position, and in the two following experiments.

The vibrations, in moving the weights from the midway to the positive position, were so small, that it was thought not worth while to observe the time of the vibration. When the weights were returned to the midway position, I determined the time of the arm's coming to the middle point of each vibration, in order to see how nearly the times of the different vibrations agreed together. In great part of the following experiments, I contented myself with observing the time of its coming to the middle point of only the first and last vibration.

EXPERIMENT II.   Aug. 6.

Weights in midway position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference
		h.	′	″		h.	′	″	
	11	10	4	0					
	11		11	0					
	11		17	0					
	11		25	0	11,				

Weights moved to positive position.

29,3									
24,1	–	–		–	26,87				
30	–	–		–	27,57				
26,2	–	–		–	28,02				
29,7	–	–		–	28,12				
26,9	–	–		–	28,05				
28,7	–	–		–	27,85				
27,1	–	–		–	27,82				
28,4									

Weights returned to midway position.

6									
	12	1	3	50 ⎫	–	1	4	1	
	13		4	34 ⎭					
18,5	–	–		–	12,37	–	–		14 52
	13		18	29 ⎫	–		18	53	
	12		19	18 ⎭					
6,5	–	–		–	11,67	–	–		14 46
	11		33	48 ⎫	–		33	39	
	12		34	51 ⎭					
15,2	–	–		–	11	–	–		13 46
	13		45	8 ⎫	–		47	25	
	12		46	22 ⎭					

Extreme points.	Divisions.	Time.	Point of rest.	Time of mid. of vibration.	Difference.
		h. ′ ″		h. ′ ″	′ ″
7,1	-	- - -	10 75	- - -	15 25
	11	2 3 48 }			
	12	5 18 }		2 2 50	
13,6					

Motion of arm on moving weights from

$$\text{midway to pos.} \quad - \quad = 15{,}87$$
$$\text{pos. to midway} \quad - \quad = 15{,}45$$

Time of one vibration - - - $= 14' \, 42''$

EXPERIMENT III.   Aug. 7.

The weights being in the positive position, and the arm a little in motion.

31,5				
29	-	- -	30,12	
31	-	- -	30,02	
29, 1				

Weights moved to midway position.

9				
	14	10 34 18 }		10 34 55
	15	35 8 }		
20,5	-	- -	14,8	- - 14 44
	15	49 31 }		49 39
	14	50 27 }		
9,2	-	- -	14,07	- - 14 38
	14	11 5 7 }		11 4 17
	15	6 18 }		

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	,	”		h.	,	‘	,	”
17,4	-	-		-	13,52	-		-	14	47
	14	11	18	46 }		11	19	4		
	13		19	58 }	-					
10,1	-	-		-	13,3	-		-	14	27
	13		33	46 }			33	31		
	14		35	26 }	-					
15,6										

Weights moved to positive position.

32					-					
	28	0	2	48 }		0	2	59		
	27		3	56 }	-					
23,7	-	-		-	27,8					
31,8	-	-		-	28,27					
25,8	-	-		-	28,62					
	27		44	58 }			47	40		
	28		46	50 }	-					
31,1										

Motion of the arm on moving weights from

pos. to mid.   -   = 15,22

mid. to pos.   -   = 14,5

Time of one vibration, when in mid. position = 14′ 39″

pos. position = 14 54

These experiments are sufficient to shew, that the attraction of the weights on the balls is very sensible, and are also sufficiently regular to determine the quantity of this attraction pretty nearly, as the extreme results do not differ from each other by more than $\frac{1}{10}$ part. But there is a circumstance in them, the reason of which does not readily appear, namely, that the effect of the attraction seems to increase, for half an

hour, or an hour, after the motion of the weights ; as it may be observed, that in all three experiments, the mean position kept increasing for that time, after moving the weights to the positive position ; and kept decreasing, after moving them from the positive to the midway position.

The first cause which occurred to me was, that possibly there might be a want of elasticity, either in the suspending wire, or something it was fastened to, which might make it yield more to a given pressure, after a long continuance of that pressure, than it did at first.

To put this to the trial, I moved the index so much, that the arm, if not prevented by the sides of the case, would have stood at about 50 divisions, so that, as it could not move farther than to 35 divisions, it was kept in a position 15 divisions distant from that which it would naturally have assumed from the stiffness of the wire ; or, in other words, the wire was twisted 15 divisions. After having remained two or three hours in this position, the index was moved back, so as to leave the arm at liberty to assume its natural position.

It must be observed, that if a wire is twisted only a little more than its elasticity admits of, then, instead of setting, as it is called, or acquiring a permanent twist all at once, it sets gradually, and, when it is left at liberty, it gradually loses part of that set which it acquired ; so that if, in this experiment, the wire, by having been kept twisted for two or three hours, had gradually yielded to this pressure, or had begun to set, it would gradually restore itself, when left at liberty, and the point of rest would gradually move backwards ; but, though the experiment was twice repeated, I could not perceive any such effect.

The arm was next suspended by a stiffer wire.

EXPERIMENT IV.   Aug. 12.
Weights in midway position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.				Difference.
		h. ' "				h. ' "				' "
	21,6	9 30 0								
	21,5	52 0								
	21,5	10 13 0			21,5					

Weights moved from midway to positive position.

27,2										
22,1		-	-	-	24,6					
27		-	-	-	24,67					
22,6		-	-	-	24,75					
26,8		-	-	-	24,8					
23,0		-	-	-	24,85					
26,6		-	-	-	24,9					
23,4		-								

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.				Difference.
15										
	17	19 25 }								
	19	20 41 }			-	10 20 31				
22,4		-	-	-	18,72	- -				7 0
	20	26 45 }								
	19	27 22 }			-	27 31				
15,1		-	-	-	18,52	- -				6 57
	19	35 1 }								
	20	48 }			-	34 28				
21,5		-	-	-	18,35	- -				7 23
	20	40 23 }								
	19	41 18 }			-	41 51				
15,3		-	-	-	18,22	- -				6 48
	18	48 36 }								
	19	49 24 }			-	48 39				
20,8		-	-	-	18,1	- -				6 58
	19	54 45 }								
	18	55 45 }			-	55 37				
15,5										

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	′	″		h.	′	″	′	″
31,3										
	25	11	10	25 ⎫	-	11	10	40		
	23		11	3 ⎬						
17,1	-	-	-	-	24,02	-	-	-	7	3
	22		17	6 ⎫	-		17	43		
	23			26 ⎬						
30,6	-	-	-	-	24,17	-	-	-	7	1
	25		24	33 ⎫	-		24	44		
	23		25	17 ⎬						
18,4	-	-	-	-	24,32	-	-	-	7	5
	23		31	21 ⎫	-		31	49		
	25		32	9 ⎬						
29,9	-	-	-	-	24,4	-	-	-	6	59
	25		38	39 ⎫	-		38	48		
	23		39	31 ⎬						
19,4	-	-	-	-	24,5	-	-	-	7	6
	23		45	16 ⎫	-		45	54		
	25		46	12 ⎬						
29,3										

Motion of arm on moving weights from

$$\text{midway to pos.} \quad - \quad = 3,1$$
$$\text{pos. to neg.} \quad - \quad - \quad = 6,18$$
$$\text{neg. to pos.} \quad - \quad = 5,92$$

Time of one vibration in neg. position $\quad - \quad = 7' 1''$

$$\text{pos. position} \quad - \quad = 7\ 3$$

EXPERIMENT V.   Aug. 20.

The weights being in the positive position, the arm was made to vibrate, by moving the index.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	′	″		h.	′	″	′	″
29,6										
21,1	-	-	-		25,2					
29,	-	-	-		25,17					
21,6										

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
22,6										
	20	10	22	47 }	-	10	23	11		
	19		23	30 }						
16,3	-	-	-		19,27					
21,9	-	-	-		19,15					
16,5	-	-	-		19,1					
21,5	-	-	-		19,07					
16,8	-	-	-		19,07					
21,2	-	-	-		19,07					
17,1	-	-	-		19,05					
20,8	-	-	-		19,02					
17,4	-	-	-		19,05					
20,6	-	-	-		19,02					
	20	11	32	16 }	-	11	33	53		
	19		33	58 }						
17,5	-	-	-		18,97				7	13
	19		41	16 }	-		41	6		
	20		43	0 }						
20,3										

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	′	″		h.	′	″	′	″
20,2										
	24	11	49	10 ⎫	–	11	49	37	7	7
	26		50	19 ⎭						
29,4		–	–	–	24,95					
	26		56	15 ⎫	–		56	44		
	25			47 ⎭						
20,8	–	–	–	–	24,92					
28,7	–	–	–	–	24,87					
21,3	–	–	–	–	24,85					
28,1	–	–	–	–	24,75					
21,5	–	–	–	–	24,67					
27,6	–	–	–	–	24,67					
22	–	–	–	–	24,7					
	24	0	45	48 ⎫	–	0	46	21	7	1
	25		46	43 ⎭						
27,2	–	–	–	–	24,7					
	25		53	11 ⎫	–		53	22		
	24		54	9 ⎭						
22,4										

Motion of arm on moving weights from

pos. to neg.   – – – $= 5,9$

neg. to pos.   – – – $= 5,98$

Time of one vibration, when weights are in

neg. position   – – – $= 7' 5''$

pos. position   – – – $= 7\ 5$

In the fourth experiment, the effect of the weights seemed to increase on standing, in all three motions of the weights, conformably to what was observed with the former wire; but, in

MDCCXCVIII.   3 R

the last experiment, the case was different; for though, on moving the weights from positive to negative, the effect seemed to increase on standing, yet, on moving them from negative to positive, it diminished.

My next trials were, to see whether this effect was owing to magnetism. Now, as it happened, the case in which the arm was inclosed, was placed nearly parallel to the magnetic east and west, and therefore, if there was any thing magnetic in the balls and weights, the balls would acquire polarity from the earth; and the weights also, after having remained some time, either in the positive or negative position, would acquire polarity in the same direction, and would attract the balls; but, when the weights were moved to the contrary position, that pole which before pointed to the north, would point to the south, and would repel the ball it was approached to; but yet, as repelling one ball towards the south has the same effect on the arm as attracting the other towards the north, this would have no effect on the position of the arm. After some time, however, the poles of the weight would be reversed, and would begin to attract the balls, and would therefore produce the same kind of effect as was actually observed.

To try whether this was the case, I detached the weights from the upper part of the copper rods by which they were suspended, but still retained the lower joint, namely, that which passed through them; I then fixed them in their positive position, in such manner, that they could turn round on this joint, as a vertical axis. I also made an apparatus, by which I could turn them half way round, on these vertical axes, without opening the door of the room.

Having suffered the apparatus to remain in this manner for

a day, I next morning observed the arm, and, having found it to be stationary, turned the weights half way round on their axes, but could not perceive any motion in the arm. Having suffered the weights to remain in this position for about an hour, I turned them back into their former position, but without its having any effect on the arm. This experiment was repeated on two other days, with the same result.

We may be sure, therefore, that the effect in question could not be produced by magnetism in the weights; for, if it was, turning them half round on their axes, would immediately have changed their magnetic attraction into repulsion, and have produced a motion in the arm.

As a further proof of this, I took off the leaden weights, and in their room placed two 10-inch magnets; the apparatus for turning them round being left as it was, and the magnets being placed horizontal, and pointing to the balls, and with their north poles turned to the north; but I could not find that any alteration was produced in the place of the arm, by turning them half round; which not only confirms the deduction drawn from the former experiment, but also seems to shew, that in the experiments with the iron rods, the effect produced could not be owing to magnetism.

The next thing which suggested itself to me was, that possibly the effect might be owing to a difference of temperature between the weights and the case; for it is evident, that if the weights were much warmer than the case, they would warm that side which was next to them, and produce a current of air, which would make the balls approach nearer to the weights. Though I thought it not likely that there should be sufficient difference, between the heat of the weights and case, to have

3 R 2

any sensible effect, and though it seemed improbable that, in all the foregoing experiments, the weights should happen to be warmer than the case, I resolved to examine into it, and for this purpose removed the apparatus used in the last experiments, and supported the weights by the copper rods, as before; and, having placed them in the midway position, I put a lamp under each, and placed a thermometer with its ball close to the outside of the case, near that part which one of the weights approached to in its positive position, and in such manner that I could distinguish the divisions by the telescope. Having done this, I shut the door, and some time after moved the weights to the positive position. At first, the arm was drawn aside only in its usual manner; but, in half an hour, the effect was so much increased, that the arm was drawn 14 divisions aside, instead of about three, as it would otherwise have been, and the thermometer was raised near $1°\frac{1}{2}$; namely, from $61°$ to $62°\frac{1}{2}$. On opening the door, the weights were found to be no more heated, than just to prevent their feeling cool to my fingers.

As the effect of a difference of temperature appeared to be so great, I bored a small hole in one of the weights, about three-quarters of an inch deep, and inserted the ball of a small thermometer, and then covered up the opening with cement. Another small thermometer was placed with its ball close to the case, and as near to that part to which the weight was approached as could be done with safety; the thermometers being so placed, that when the weights were in the negative position, both could be seen through one of the telescopes, by means of light reflected from a concave mirror.

EXPERIMENT VI.   Sept. 6.

Weights in midway position.

Extreme points.	Divisions.	Time.		Point of rest.	Thermometer	
		h	′		in Air.	in Weight.
18,9	9	43	–	55,5	
18,85	10	3	18,85		

Weights moved to negative position.

13,1	–	10	12	–	55,5	55,8
18,4	–		18	15,82		
13,4	–		25			
missed.						
13,6	–		39	–	55,5	55,8
17,6	–		46	15,65		
13,8	–		53	15,65		
17,4	–	11	0	15,65		
14,0	–		7	15,65		
17,2	–		14	–	55,5	

Weights moved to positive position.

25,8	–		23			
17,5	–		30	21,55		
25,4	–		37	21,6		
18,1	–		44	21,65		
25,0	–		51			
missed.						
24,7	–	0	5	–		
19,	–		12	21,77		
24,4	–		19			

Motion of arm on moving weights from midway to $-$ = 3,03

$-$ to $+$ = 5,9

EXPERIMENT VII.   Sept. 18.

Weights in midway position.

Extreme points.	Divisions.	Time.	Point of rest.	Thermometer in Air.	inWeight.
		h '			
	19,4	8 30	-	56,7	
	19,4	9 32	-	56,6	

Weights moved to negative position.

13,6	-	40	-	-	57,2
18,8	-	47	16,25		
13,8	-	54			

Eight extreme points missed.

16,9	-	10 58			
14,5	-	11 5	15,62		
16,6	-	12			

Weights moved to positive position.

26,4	-	20	-	56,5	
17,2	-	28	21,72		
26,1	-	35			

Four extreme points missed.

19,3	-	0 10			
25,1	-	17	22,3		
19,7	-	24			

Motion of arm on moving weights from midway to $-\ = 3,15$

$-$ to $+ = 6,1$.

EXPERIMENT VIII. Sept. 23.

Weights in midway position.

Extreme points.	Divisions.	Time.	Point of rest.	Thermometer in Air.	in Weight.
		h '			
19,3		9 46	-	53,1	
19,2		10 45	19,2	53,1	

Weights moved to negative position.

13,5	-	56	-		53,6
18,6	-	11 3	16,07		
13,6	-	10			

Four extreme points missed.

17,4	-	44			
14,1	-	51	15,7		
17,2	-	58	-	-	53,6

Weights moved to positive position.

15,7	-	0 1			
26,7	-	8	21,42		
16,6	-	15	-	53,15	

Two extreme points missed.

25,9	-	36			
18,1	-	43	21,9		
25,5	-	50			

Motion of arm on moving weights from midway to — = 3,13

— to + = 5,72

In these three experiments, the effect of the weight appeared to increase from two to five tenths of a division, on standing an hour; and the thermometers shewed, that the weights were three or five tenths of a degree warmer than the air close to the case. In the two last experiments, I put a lamp into the room, over night, in hopes of making the air warmer than the weights, but without effect, as the heat of the weights exceeded that of the air more in these two experiments than in the former.

On the evening of October 17, the weights being placed in the midway position, lamps were put under them, in order to warm them; the door was then shut, and the lamps suffered to burn out. The next morning it was found, on moving the weights to the negative position, that they were $7°\frac{1}{2}$ warmer than the air near the case. After they had continued an hour in that position, they were found to have cooled $1°\frac{1}{2}$, so as to be only $6°$ warmer than the air. They were then moved to the positive position; and in both positions the arm was drawn aside about four divisions more, after the weights had remained an hour in that position, than it was at first.

May 22, 1798. The experiment was repeated in the same manner, except that the lamps were made so as to burn only a short time, and only two hours were suffered to elapse before the weights were moved. The weights were now found to be scarcely $2°$ warmer than the case; and the arm was drawn aside about two divisions more, after the weights had remained an hour in the position they were moved to, than it was at first.

On May 23, the experiment was tried in the same manner, except that the weights were cooled by laying ice on them; the ice being confined in its place by tin plates, which, on

moving the weights, fell to the ground, so as not to be in the way. On moving the weights to the negative position, they were found to be about 8° colder than the air, and their effect on the arm seemed now to diminish on standing, instead of increasing, as it did before; as the arm was drawn aside about $2\frac{1}{2}$ divisions less, at the end of an hour after the motion of the weights, than it was at first.

It seems sufficiently proved, therefore, that the effect in question is produced, as above explained, by the difference of temperature between the weights and case; for, in the 6th, 8th, and 9th experiments, in which the weights were not much warmer than the case, their effect increased but little on standing; whereas, it increased much, when they were much warmer than the case, and decreased much, when they were much cooler.

It must be observed, that in this apparatus, the box in which the balls play is pretty deep, and the balls hang near the bottom of it, which makes the effect of the current of air more sensible than it would otherwise be, and is a defect which I intend to rectify in some future experiments.

EXPERIMENT IX.   April 29.

Weights in positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of middle of vibration.		
		h	'	"		h	'	"
34,7								
35	-	-	-	-	34,84			
34,65								

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	′	″		h.	′	″	′	′
23,8										
	28	11	18	29 ⎫	–	11	18	43		
	29			58 ⎭						
33,2	–	–		–	28,52					
	29		25	27 ⎫	–		25	40		
	28			57 ⎭						
23,9	–	–		–	28,25					
32	–	–		–	28,01					
24,15	–	–		–	27,82					
31	–	–		–	27,63					
24,4	–	–		–	27,55					
30,4	–	–		–	27,47					
	28	0	7	4 ⎫	–	0	7	26		
	27			53 ⎭						
24,7										

Motion of arm   –   = 6,32

Time of vibration   –   = 6′ 58″

EXPERIMENT X.   May 5.

Weights in positive position.

34,5				
33,5	–	–	–	33,97
34,4				

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
22,3										
	28	10	43	42 ⎫	–	10	43	36		
	29		44	6 ⎭						
33,2	–	–		–	27,82	–			7	0
	28		50	33 ⎫	–		50	36		
	27		51	0 ⎭						
22,6	–	–		–	27,72					

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.			Difference.	
		h.	'	"		h.	'	"	'	"
32,5	-	-	-	-	27,7					
23,2	-	-	-	-	27,58					
31,45	-	-	-	-	27,4					
23,5	-	-	-	-	27,28					
	27	11	25	20 }	-	11	25	24		
	28			58 }						
30,7	-	-	-	-	27,21	-	-		7	3
	28		32	0 }	-		32	27		
	27		32	40 }						
23,95	-	-	-	-	27,21	-	-		6	56
	27		39	19 }	-		39	23		
	28		40	2 }						
30,25										

$$\text{Motion of arm} \quad - \quad = 6,15$$
$$\text{Time of vibration} \quad - \quad = 6' \; 59''$$

EXPERIMENT XI. May 6.

Weights in positive position.

34,9					
34,1	-	-		34,47	
34,8	-	-		34,49	
34,25					

Weights moved to negative position.

23,3							
	28	9	59	59 }	-	10	0	8
	29	10	0	27 }			
33,3	-	-	-		28,42		7	5
	29		6	52 }			
	27		7	51 }			

3 S 2

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.	
		h.	,	.		h.	, .
23,8	-	-	-		28,35		
32,5	-	-	-		28,3		
24,4 missed.							
24,8							
31,3	-	-	-		28,17		
	29	10	48	37 }	-	10	49 8
	28		49	21 }			
25,3	-	-	-		28,2		
	28		56	8 }	-		56 13
	29		56	56 }			
30,9							

Motion of arm   -   = 6,07

Time of vibration   -   = 7′ 1″

In the three foregoing experiments, the index was purposely moved so that, before the beginning of the experiment, the balls rested as near the sides of the case as they could, without danger of touching it; for it must be observed, that when the arm is at 35, they begin to touch. In the two following experiments, the index was in its usual position.

EXPERIMENT XII.   May 9.

Weights in negative position.

17,4	9	45	0	
17,4		58	0	
17,4	10	8	0	
17,4		10	0	17,4

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	,	"		h.	,	"
28,85								
	24	10	20	50 }	–	10	20	59
	22		21	46 }				
18,4	–	–		–	23,49			
28,3	–	–		–	23,57			
19,3	–	–		–	23,67			
27,8	–	–		–	23,72			
20	–	–		–	23,8			
27,4	–	–		–	23,83			
	24	11	3	13 }	–	11	3	14
	23			54 }				
20,55	–	–		–	23,87			
	23		9	45 }	–		10	18
	24		10	28 }				
27								

Motion of arm -   -   -   = 6,09
Time of vibration   -   -   = 7′ 3″

EXPERIMENT XIII.   May 25.

Weights in negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.
16						
18,3	–	–		–	17,2	
16,2						

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
29,6								
	25	10	22	22 }	–	10	22	56
	24		0	45 }				
17,4	–	–		–	23,32			
	23		29	59 }	–		30	3
	24		30	23 }				
28,9	–	–		–	23,4			
	24		36	58 }	–		37	7
	23		37	24 }				
18,4	–	–		–	23,52			

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	′	″		h.	′	″
	23	10	44	3 }	–	10	44	14
	24			31 }				
28,4	–	–		–	23,62			
19,3	–	–		–	23,7			
27,8	–	–		–	23,7			
	24	11	5	26 }	–	11	5	31
	23		6	1 }				
19,9	–	–		–	23,72			
	23		12	12 }	–		12	35
	24			50 }				
27,3								

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
13,5								
21,8	–	–		–	17,75			
	18		37	34 }	–		37	39
	17		38	10 }				
13,9	–	–		–	17,67			
	17		44	26 }	–		44	45
	18		45	4 }				
21,1	–	–		–	17,62			
14,4	–	–		–	17,6			
20,5	–	–		–	17,52			
14,7	–	–		–	17,47			
20	–	–		–	17,42			
	18	0	19	57 }	–	0	20	24
	17		20	52 }				
15	–	–		–	17,37			
	17		27	15 }	–		27	30
	18		28	15 }				
19,5								

Motion of the arm on moving weights from — to + = 6,12

+ to — = 5,97

Time of vibration at   -   -   -   -   + = 7′ 6″

— = 7 7

EXPERIMENT XIV.   May 26.

Weights in negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	′	″		h.	′	″
	16,1	9	18	0				
	16,1		24	0				
	16,1		46	0				
	16,1		49	0	16,1			

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
27,7								
	23	10	0	46 }	–	10	1	1
	22		1	16 }				
17,3	–	–	–		22,37			
	22		7	58 }	–		8	5
	23		8	27 }				
27,2	–	–	–		22,5			
	23		15	2 }	–		15	9
	22			32 }				
18,3	–	–	–		22,65			
26,8	–	–	–		22,75			
19,1	–	–	–		22,85			
26,4	–	–	–		22,97			
	23		43	40 }	–		43	32
	22		44	22 }				
20	–	–	–		23,15			
	22		49	53 }	–		50	41
	23		50	37 }				
26,2								

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
12,4								
	16	11	7	53 }	–	11	8	25
	17		8	27 }				
21,5	–	–	–		17,02			
	17		15	30 }	–		15	27
	16		16	3 }				

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	′	″		h.	′	″
12,7	-	-	-	-	16,9			
20,7	-	-		-	16,85			
13,3	-	-		-	16,82			
20	-	-		-	16,72			
13,6	-	-		-	16,67			
	16	11	50	33 ⎱	-	11	50	58
	17		51	19 ⎰				
19,5	-	-		-	16,65			
	17		57	53 ⎱	-		58	6
	16		58	44 ⎰				
14								

Motion of arm by moving weights from — to + = 6,27

+ to — = 6,13

Time of vibration at + = 7′ 6″

— = 7 6

In the next experiment, the balls, before the motion of the weights, were made to rest as near as possible to the sides of the case, but on the contrary side from what they did in the 9th, 10th, and 11th experiments.

EXPERIMENT XV.   May 27.

Weights in negative position.

3,9				
3,35	-	-	-	3,61
3,85	-	-	-	3,61
3,4				

Weights moved to positive position.

Extreme points.	Divisions.	Time. (h. ' ")	Point of rest.	Time of mid. of vibration. (h. ' ")
15,4				
	10	10 5 59 }	–	10 5 56
	9	6 27 }		
4,8	–	– –	9,95	
	9	12 43 }	–	13 5
	10	13 11 }		
14,8	–	– –	10,07	
	10	20 24 }	–	20 13
	9	56 }		
5,9	–	– –	10,23	
14,35	–	– –	10,35	
6,8	–	– –	10,46	
13,9	–	– –	10,52	
	11	48 30 }	–	48 42
	10	49 11 }		
7,5	–	– –	10,6	
	10	55 26 }	–	55 48
	11	56 10 }		
13,5				

Motion of the arm — — — 6;34

Time of vibration — — 7' 7"

The two following experiments were made by Mr. GILPIN, who was so good as to assist me on the occasion.

EXPERIMENT XVI.   May 28.

Weights in negative position.

22,55			
8,4	–	– –	15,09
21	–	– –	14,9
9,2			

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.
		h.	,	"		h. , "
26,6						
	22	10	22	53 }	–	10 23 15
	21		23	20 }		
15,8	–	–	–		21	
	20		30	7 }	–	30 30
	21			36 }		
25,8	–	–	–		21,05	
	22		37	23 }	–	37 45
	21			55 }		
16,8	–	–	–		21,11	
	20		44	29 }	–	45 1
	21		45	4 }		
25,05	–	–	–		21,11	
	22		51	54 }	–	52 20
	21		52	32 }		
17,57	–	–	–		21,2	
	21		59	31 }	–	59 34
	22	11	0	13 }		
24,6	–	–	–		21,28	
	22		6	24 }	–	11 6 49
	21		7	9 }		
18,3						

Motion of the arm   –   –   –   = 6,1
Time of vibration   –   –   –   –   = 7' 16"

EXPERIMENT XVII.   May 30.

Weights in negative position.

17,2	10 19 0		
17,1	25 0		
17,07	29 0		
17,15	40 0		
17,45	49 0		
17,42	51 0		
17,42	11 1 0	17,42	

Weights moved to positive position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	,	"		h.	,	.
28,8								
	24	11	11	23 }	-	11	11	37
	23			49 }				
18,1	-	-		-	23,2			
	22		18	13 }	-		18	42
	23			43 }				
27,8	-	-		-	23,12			
	24		25	19 }	-		25	40
	23			49 }				
18,8	-	-		-	23,2			
	23		32	41 }	-		32	43
	24		33	13 }				
27,38	-	-		-	23,31			
	24		39	28 }	-		39	44
	23		40	3 }				
19,7	-	-		-	23,44			
	23		46	33 }	-		46	46
	24		47	11 }				
27	-	-		-	23,52			
	24		53	36 }	-		53	48
	23		54	17 }				
20,4	-	-		-	23,57			
	23	0	0	34 }	-	0	0	55
	24		1	18 }				
26,5	-	-		-	23,55			
	24		7	34 }	-		7	50
	23		8	21 }				
20,8	-	-		-	23,59			
	23		14	30 }	-		14	58
	24		15	24 }				
26,25								

Weights moved to negative position.

Extreme points.	Divisions.	Time.			Point of rest.	Time of mid. of vibration.		
		h.	,	"		h.	,	"
13,3								
	17	o	32	19 }	–	o	32	44
	18			48 }				
22,4	–	–		–	17,95			
	18		39	46 }	–		39	44
	17		40	19 }				
13,7	–	–		–	17,85			
	17		46	26 }	–		46	48
	18		47	0 }				
21,6	–	–		–	17,72			
	18		53	43 }	–		53	50
	17		54	20 }				
14	–	–		–	17,6			
	17	1	0	39 }	–	1	0	55
	18		1	20 }				
20,8	–	–		–	17,47			
	18		7	39 }	–		7	59
	17		8	21 }				
14,3	–	–		–	17,37			
	17		14	54 }	–		15	4
	18		15	42 }				
20,1	–	–		–	17,27			
	18		21	32 }	–		22	5
	17		22	22 }				
14,6								

Motion of the arm on moving weights from — to + = 5,78

+ to — = 5,64

Time of vibration at   -   -   + = 7′ 2′

— = 7 3

On the Method of computing the Density of the Earth from these Experiments.

I shall first compute this, on the supposition that the arm and copper rods have no weight, and that the weights exert no sensible attraction, except on the nearest ball; and shall then examine what corrections are necessary, on account of the arm and rods, and some other small causes.

The first thing is, to find the force required to draw the arm aside, which, as was before said, is to be determined by the time of a vibration.

The distance of the centres of the two balls from each other is 73,3 inches, and therefore the distance of each from the centre of motion is 36,65, and the length of a pendulum vibrating seconds, in this climate, is 39,14; therefore, if the stiffness of the wire by which the arm is suspended is such, that the force which must be applied to each ball, in order to draw the arm aside by the angle A, is to the weight of that ball as the arch of A to the radius, the arm will vibrate in the same time as a pendulum whose length is 36,65 inches, that is, in $\sqrt{\frac{36,65}{39,14}}$ seconds; and therefore, if the stiffness of the wire is such as to make it vibrate in N seconds, the force which must be applied to each ball, in order to draw it aside by the angle A, is to the weight of the ball as the arch of $A \times \frac{1}{N^2} \times \frac{36,65}{39,14}$ to the radius. But the ivory scale at the end of the arm is 38,3 inches from the centre of motion, and each division is $\frac{1}{20}$ of an inch, and therefore subtends an angle at the centre, whose arch is $\frac{1}{766}$; and therefore the force which must be applied to each ball, to draw the

arm aside by one division, is to the weight of the ball as $\frac{1}{766\,N^2}\frac{36,65}{39,14}$ to 1, or as $\frac{1}{818\,N^2}$ to 1.

The next thing is, to find the proportion which the attraction of the weight on the ball bears to that of the earth thereon, supposing the ball to be placed in the middle of the case, that is, to be not nearer to one side than the other. When the weights are approached to the balls, their centres are 8,85 inches from the middle line of the case; but, through inadvertence, the distance, from each other, of the rods which support these weights, was made equal to the distance of the centres of the balls from each other, whereas it ought to have been somewhat greater. In consequence of this, the centres of the weights are not exactly opposite to those of the balls, when they are approached together; and the effect of the weights, in drawing the arm aside, is less than it would otherwise have been, in the triplicate ratio of $\frac{8,85}{36,65}$ to the chord of the angle whose sine is $\frac{8,85}{36,65}$, or in the triplicate ratio of the cosine of $\frac{1}{2}$ this angle to the radius, or in the ratio of ,9779 to 1.

Each of the weights weighs 2439000 grains, and therefore is equal in weight to 10,64 spherical feet of water; and therefore its attraction on a particle placed at the centre of the ball, is to the attraction of a spherical foot of water on an equal particle placed on its surface, as $10,64 \times ,9779 \times \overline{\frac{6}{8,85}}\Big|^2$ to 1. The mean diameter of the earth is 41800000 feet;* and therefore, if the mean density of the earth is to that of water as D to one, the attraction of the leaden weight on the ball will be to that

---

* In strictness, we ought, instead of the mean diameter of the earth, to take the diameter of that sphere whose attraction is equal to the force of gravity in this climate; but the difference is not worth regarding.

of the earth thereon, as $10,64 \times ,9779 \times \overline{\frac{6}{8,85}}\Big|^2$ to 41800000 D
$:: 1$ to 8739000 D.

It is shewn, therefore, that the force which must be applied to each ball, in order to draw the arm one division out of its natural position, is $\frac{1}{818\,N^2}$ of the weight of the ball; and, if the mean density of the earth is to that of water as D to 1, the attraction of the weight on the ball is $\frac{1}{8739000\,D}$ of the weight of that ball; and therefore the attraction will be able to draw the arm out of its natural position by $\frac{818\,N^2}{8739000\,D}$ or $\frac{N^2}{10683\,D}$ divisions; and therefore, if on moving the weights from the midway to a near position the arm is found to move B divisions, or if it moves 2 B divisions on moving the weights from one near position to the other, it follows that the density of the earth, or D, is $\frac{N^2}{10683\,B}$.

We must now consider the corrections which must be applied to this result; first, for the effect which the resistance of the arm to motion has on the time of the vibration: 2d, for the attraction of the weights on the arm: 3d, for their attraction on the farther ball: 4th, for the attraction of the copper rods on the balls and arm: 5th, for the attraction of the case on the balls and arm: and 6th, for the alteration of the attraction of the weights on the balls, according to the position of the arm, and the effect which that has on the time of vibration. None of these corrections, indeed, except the last, are of much signification, but they ought not entirely to be neglected.

As to the first, it must be considered, that during the vibrations of the arm and balls, part of the force is spent in accele-

rating the arm; and therefore, in order to find the force required to draw them out of their natural position, we must find the proportion which the forces spent in accelerating the arm and balls bear to each other.

Let EDC $e\,d\,c$ (fig. 4.) be the arm. B and b the balls. Cs the suspending wire. The arm consists of 4 parts; first, a deal rod D $c\,d$, 73,3 inches long; 2d, the silver wire DC d, weighing 170 grains; 3d, the end pieces DE and $e\,d$, to which the ivory vernier is fastened, each of which weighs 45 grains; and 4th, some brass work C c, at the centre. The deal rod, when dry, weighs 2320 grains, but when very damp, as it commonly was during the experiments, weighs 2400; the transverse section is of the shape represented in fig. 5; the thickness BA, and the dimensions of the part DE $e\,d$, being the same in all parts; but the breadth B b diminishes gradually, from the middle to the ends. The area of this section is ,33 of a square inch at the middle, and ,146 at the end; and therefore, if any point x (fig. 4.) is taken in $c\,d$, and $\frac{c\,x}{c\,d}$ is called x, this rod weighs $\frac{2400 \times ,33}{73,3 \times ,238}$ per inch at the middle; $\frac{2400 \times ,146}{73,3 \times ,238}$ at the end, and $\frac{2400}{73,3} \times \frac{,33 - ,184\,x}{,238}$ $= \frac{3320 - 1848\,x}{73,3}$ at x; and therefore, as the weight of the wire is $\frac{170}{73,3}$ per inch, the deal rod and wire together may be considered as a rod whose weight at $x = \frac{3490 - 1848\,x}{73,3}$ per inch.

But the force required to accelerate any quantity of matter placed at x, is proportional to x^2; that is, it is to the force required to accelerate the same quantity of matter placed at d as x^2 to 1; and therefore, if $c\,d$ is called l, and x is supposed to flow, the fluxion of the force required to accelerate the deal rod

and wire is proportional to $\frac{x^2 l \dot{x} \times 3490 - 1848\,x}{73,3}$, the fluent of which,

generated while x flows from c to d, $= \frac{l}{73,3} \times \frac{3490}{3} - \frac{1848}{4} = 350$;

so that the force required to accelerate each half of the deal rod and wire, is the same as is required to accelerate 350 grains placed at d.

The resistance to motion of each of the pieces de, is equal to that of 48 grains placed at d; as the distance of their centres of gravity from C is 38 inches. The resistance of the brass work at the centre may be disregarded; and therefore the whole force required to accelerate the arm, is the same as that required to accelerate 398 grains placed at each of the points D and d.

Each of the balls weighs 11262 grains, and they are placed at the same distance from the centre as D and d; and therefore, the force required to accelerate the balls and arm together, is the same as if each ball weighed 11660, and the arm had no weight; and therefore, supposing the time of a vibration to be given, the force required to draw the arm aside, is greater than if the arm had no weight, in the proportion of 11660 to 11262, or of 1,0353 to 1.

To find the attraction of the weights on the arm, through d draw the vertical plane $d\,w\,b$ perpendicular to Dd, and let w be the centre of the weight, which, though not accurately in this plane, may, without sensible error, be considered as placed therein, and let b be the centre of the ball; then wb is horizontal and $= 8,85$, and $d\,b$ is vertical and $= 5,5$; let $w\,a = a$, $w\,b = b$, and let $\frac{d\,x}{d\,c}$, or $1 - x = z$; then the attraction of the weight on a particle of matter at x, in the direction $d\,w$, is to its attraction on the same particle placed at b :: $b^3 : \overline{a^2 + z^2\,l^2}^{\frac{3}{2}}$, or is pro-

portional to $\dfrac{b^3}{\overline{a^2 + z^2\ l^2}\vert^{\frac{3}{2}}}$, and the force of that attraction to move

the arm, is proportional to $\dfrac{b^3 \times \overline{1-z}}{\overline{a^2 + z^2\ l^2}\vert^{\frac{3}{2}}}$, and the weight of the

deal rod and wire at the point x, was before said to be

$\dfrac{3490 - 1848\ x}{73,3} = \dfrac{1642 + 1848\ z}{73,3}$ per inch; and therefore, if $d\,x$ flows,

the fluxion of the power to move the arm $= l\,\dot z \times \dfrac{1642 + 1848\ z}{73,3}$

$+ \dfrac{b^3 \times \overline{1-z}}{\overline{a^2 + z^2\ l^2}\vert^{\frac{3}{2}}} = \dot z \times \overline{821 + 924 z} \times \dfrac{b^3 \times \overline{1-z}}{\overline{a^2 + l^2\ z^2}\vert^{\frac{3}{2}}} = \dfrac{b^3\ \dot z \times \overline{821 + 103\,z - 924\,z^2}}{\overline{a^2 + l^2\ z^2}\vert^{\frac{3}{2}}}$

$= \dfrac{b^3\ z \times 821 + 103 z + \dfrac{924\ a^2}{l^2}}{\overline{a^2 + l^2\ z^2}\vert^{\frac{3}{2}}} - \dfrac{924\ b^3\ \dot z \times \dfrac{a^2}{l^2} + z^2}{\overline{a^2 + l^2\ z^2}\vert^{\frac{3}{2}}}$; which, as $\dfrac{a^2}{l^2} = ,08 =$

$\dfrac{b^3\ \dot z \times \overline{895 + 103\ z}}{\overline{a^2 + l^2\ z^2}\vert^{\frac{3}{2}}} - \dfrac{924\ b^3\ \dot z}{l^2\surd \overline{a^2 + l^2\ z^2}}$.   The fluent of this $= \dfrac{895\ b^3\ z}{a^2 \surd \overline{a^2 + l^2\ z^2}}$

$- \dfrac{103\ b^3}{l^2 \surd \overline{a^2 + l^2\ z^2}} + \dfrac{103\ b^3}{l^2\ a} - \dfrac{924\ b^3}{l^3}$ log. $\dfrac{l\,z + \surd \overline{a^2 + l^2\ z^2}}{a}$, and the force

with which the attraction of the weight, on the nearest half of

the deal rod and wire, tends to move the arm, is proportional

to this fluent generated while z flows from o to 1, that is, to

128 grains.

The force with which the attraction of the weight on the

end-piece de tends to move the arm, is proportional to $47 \times \dfrac{b^3}{a^3}$,

or 29 grains; and therefore the whole power of the weight to

move the arm, by means of its attraction on the nearest part

thereof, is equal to its attraction on 157 grains placed at b,

which is $\dfrac{157}{11260}$, or ,0139 of its attraction on the ball.

It must be observed, that the effect of the attraction of the

weight on the whole arm is rather less than this, as its attrac-

tion on the farther half draws it the contrary way; but, as the

attraction on this is small, in comparison of its attraction on the

nearer half, it may be disregarded.

The attraction of the weight on the furthest ball, in the direction bw, is to its attraction on the nearest ball $: : wd^3 : wD^3$ $: : ,0017 : 1$; and therefore the effect of the attraction of the weight on both balls, is to that of its attraction on the nearest ball $: : ,9983 : 1$.

To find the attraction of the copper rod on the nearest ball, let b and w (fig. 6.) be the centres of the ball and weight, and ea the perpendicular part of the copper rod, which consists of two parts, ad and de. ad weighs 22000 grains, and is 16 inches long, and is nearly bisected by w. de weighs 41000, and is 46 inches long. wb is 8,85 inches, and is perpendicular to ew. Now, the attraction of a line ew, of uniform thickness, on b, in the direction bw, is to that of the same quantity of matter placed at $w : : bw : eb$; and therefore the attraction of the part da equals that of $\frac{22000 \times wb}{db}$, or 16300, placed at w; and the attraction of de equals that of $41000 \times \frac{ew}{ed} \times \frac{bw}{be} - 41000 \times \frac{dw}{ed} \times \frac{bw}{bd}$, or 2500, placed at the same point; so that the attraction of the perpendicular part of the copper rod on b, is to that of the weight thereon, as 18800 : 2439000, or as ,00771 to 1. As for the attraction of the inclined part of the rod and wooden bar, marked Pr and rr in fig. 1, it may safely be neglected, and so may the attraction of the whole rod on the arm and farthest ball; and therefore the attraction of the weight and copper rod, on the arm and both balls together, exceeds the attraction of the weight on the nearest ball, in the proportion of ,9983 + ,0139 + ,0077 to one, or of 1,0199 to 1.

The next thing to be considered, is the attraction of the mahogany case. Now it is evident, that when the arm stands at

the middle division, the attractions of the opposite sides of the case balance each other, and have no power to draw the arm either way. When the arm is removed from this division, it is attracted a little towards the nearest side, so that the force required to draw the arm aside is rather less than it would otherwise be; but yet, if this force is proportional to the distance of the arm from the middle division, it makes no error in the result; for, though the attraction will draw the arm aside more than it would otherwise do, yet, as the accelerating force by which the arm is made to vibrate is diminished in the same proportion, the square of the time of a vibration will be increased in the same proportion as the space by which the arm is drawn aside, and therefore the result will be the same as if the case exerted no attraction; but, if the attraction of the case is not proportional to the distance of the arm from the middle point, the ratio in which the accelerating force is diminished is different in different parts of the vibration, and the square of the time of a vibration will not be increased in the same proportion as the quantity by which the arm is drawn aside, and therefore the result will be altered thereby.

On computation, I find that the force by which the attraction draws the arm from the centre is far from being proportional to the distance, but the whole force is so small as not to be worth regarding; for, in no position of the arm does the attraction of the case on the balls exceed that of $\frac{1}{5}$th of a spheric inch of water, placed at the distance of 1 inch from the centre of the balls; and the attraction of the leaden weight equals that of 10,6 spheric feet of water placed at 8,85 inches, or of 234 spheric inches placed at 1 inch distance; so that the attraction of the case on the balls can in no position of the arm exceed

$\frac{1}{1170}$ of that of the weight. The computation is given in the Appendix.

It has been shewn, therefore, that the force required to draw the arm aside one division, is greater than it would be if the arm had no weight, in the ratio of 1,0353 to 1, and therefore $= \frac{1,0353}{818\,N^2}$ of the weight of the ball; and moreover, the attraction of the weight and copper rod on the arm and both balls together, exceeds the attraction of the weight on the nearest ball, in the ratio of 1,0199 to 1, and therefore $= \frac{1,0199}{8739000\,D}$ of the weight of the ball; consequently D is really equal to $\frac{818\,N^2}{1,0353}$ $\times \frac{1,0199}{8739000\,B}$, or $\frac{N^2}{10844\,B}$, instead of $\frac{N^2}{10683\,B}$, as by the former computation. It remains to be considered how much this is affected by the position of the arm.

Suppose the weights to be approached to the balls; let W, (fig. 7.) be the centre of one of the weights; let M be the centre of the nearest ball at its mean position, as when the arm is at 20 divisions; let B be the point which it actually rests at; and let A be the point which it would rest at, if the weight was removed; consequently, AB is the space by which it is drawn aside by means of the attraction; and let Mβ be the space by which it would be drawn aside, if the attraction on it was the same as when it is at M. But the attraction at B is greater than at M, in the proportion of $WM^2 : WB^2$; and therefore, $AB = M\beta \times \frac{WM^2}{WB^2} = M\beta \times 1 + \frac{2MB}{MW}$, very nearly.

Let now the weights be moved to the contrary near position, and let w be now the centre of the nearest weight, and b the point of rest of the centre of the ball; then $Ab = M\beta \times 1 + \frac{2Mb}{MW}$, and $Bb = M\beta \times 2 + \frac{2Mb}{MW} + \frac{2MB}{MW} = 2M\beta \times 1 + \frac{Bb}{MW}$; so

that the whole motion Bb is greater than it would be if the attraction on the ball was the same in all places as it is at M, in the ratio of $1 + \frac{Bb}{MW}$ to one; and, therefore, does not depend sensibly on the place of the arm, in either position of the weights, but only on the quantity of its motion, by moving them.

This variation in the attraction of the weight, affects also the time of vibration; for, suppose the weights to be approached to the balls, let W be the centre of the nearest weight; let B and A represent the same things as before; and let x be the centre of the ball, at any point of its vibration; let AB represent the force with which the ball, when placed at B, is drawn towards A by the stiffness of the wire; then, as B is the point of rest, the attraction of the weight thereon will also equal AB; and, when the ball is at x, the force with which it is drawn towards A, by the stiffness of the wire, $= A x$, and that with which it is drawn in the contrary direction, by the attraction, $= AB \times \frac{WB^2}{Wx^2}$; so that the actual force by which it is drawn towards $A = A x - \frac{AB \times WB^2}{Wx^2} = AB + Bx - \overline{AB \times 1 + \frac{2Bx}{WB}} = Bx - \frac{2Bx \times AB}{WB}$, very nearly. So that the actual force with which the ball is drawn towards the middle point of the vibration, is less than it would be if the weights were removed, in the ratio of $1 - \frac{2AB}{WB}$ to one, and the square of the time of a vibration is increased in the ratio of 1 to $1 - \frac{2AB}{WB}$; which differs very little from that of $1 + \frac{Bb}{MW}$ to 1, which is the ratio in which the motion of the arm, by moving the weights from one near position to the other, is increased.

The motion of the ball answering to one division of the arm $= \frac{36,35}{20 \times 38,3}$; and, if $m\,\mathrm{B}$ is the motion of the ball answering to d divisions on the arm, $\frac{\mathrm{MB}}{\mathrm{WM}} = \frac{36,35\,d}{20 \times 38,3 \times 8,85} = \frac{d}{185}$; and therefore, the time of vibration, and motion of the arm, must be corrected as follows :

If the time of vibration is determined by an experiment in which the weights are in the near position, and the motion of the arm, by moving the weights from the near to the midway position, is d divisions, the observed time must be diminished in the subduplicate ratio of $1 - \frac{2d}{185}$ to 1, that is, in the ratio of $1 - \frac{d}{185}$ to 1; but, when it is determined by an experiment in which the weights are in the midway position, no correction must be applied.

To correct the motion of the arm caused by moving the weights from a near to the midway position, or the reverse, observe how much the position of the arm differs from 20 divisions, when the weights are in the near position: let this be n divisions, then, if the arm at that time is on the same side of the division of 20 as the weight, the observed motion must be diminished by the $\frac{2n}{185}$ part of the whole; but, otherwise, it must be as much increased.

If the weights are moved from one near position to the other, and the motion of the arm is $2d$ divisions, the observed motion must be diminished by the $\frac{2d}{185}$ part of the whole.

If the weights are moved from one near position to the other, and the time of vibration is determined while the weights are in one of those positions, there is no need of correcting either the motion of the arm, or the time of vibration.

CONCLUSION.

The following Table contains the Result of the Experiments.

Exper.	Mot. weight	Mot. arm	Do. corr.	Time vib.	Do. corr.	Density.
1 {	m. to +	14,32	13,42	′ ″	-	5,5
	+ to m.	14,1	13,17	14,55	-	5,61
2 {	m. to +	15,87	14,69	-	-	4,88
	+ to m.	15,45	14,14	14,42	-	5,07
3 {	+ to m.	15,22	13,56	14,39	-	5,26
	m. to +	14,5	13,28	14,54	-	5,55
4 {	m. to +	3,1	2,95		6,54	5,36
	+ to —	6,18	-	7,1	-	5,29
	— to +	5,92	-	7,3	-	5,58
5 {	+ to —	5,9	-	7,5	-	5,65
	— to +	5,98	-	7,5	-	5,57
6 {	m. to —	3,03	2,9	-	-	5,53
	— to +	5,9	5,71		-	5,62
7 {	m. to —	3,15	3,03	7,4 by mean.	6,57	5,29
	— to +	6,1	5,9			5,44
8 {	m. to —	3,13	3,00		-	5,34
	— to +	5,72	5,54	-	-	5,79
9	+ to —	6,32	-	6,58	-	5,1
10	+ to —	6,15	-	6,59	-	5,27
11	+ to —	6,07	-	7,1	-	5,39
12	— to +	6,09	-	7,3	-	5,42
13 {	— to +	6,12	-	7,6	-	5,47
	+ to —	5,97	-	7,7	-	5,63
14 {	— to +	6,27	-	7,6	-	5,34
	+ to —	6,13	-	7,6	-	5,46
15	— to +	6,34	-	7,7	-	5,3
16	— to +	6,1	-	7,16	-	5,75
17 {	— to +	5,78	-	7,2	-	5,68
	+ to —	5,64	-	7,3	-	5,85

From this table it appears, that though the experiments agree pretty well together, yet the difference between them, both in the quantity of motion of the arm and in the time of vibration, is greater than can proceed merely from the error of observation. As to the difference in the motion of the arm, it may very well be accounted for, from the current of air produced by the difference of temperature; but, whether this can account for the difference in the time of vibration, is doubtful. If the current of air was regular, and of the same swiftness in all parts of the vibration of the ball, I think it could not; but, as there will most likely be much irregularity in the current, it may very likely be sufficient to account for the difference.

By a mean of the experiments made with the wire first used, the density of the earth comes out $5,48$ times greater than that of water; and by a mean of those made with the latter wire, it comes out the same; and the extreme difference of the results of the 23 observations made with this wire, is only $,75$; so that the extreme results do not differ from the mean by more than $,38$, or $\frac{1}{14}$ of the whole, and therefore the density should seem to be determined hereby, to great exactness. It, indeed, may be objected, that as the result appears to be influenced by the current of air, or some other cause, the laws of which we are not well acquainted with, this cause may perhaps act always, or commonly, in the same direction, and thereby make a considerable error in the result. But yet, as the experiments were tried in various weathers, and with considerable variety in the difference of temperature of the weights and air, and with the arm resting at different distances from the sides of the case, it seems very unlikely that this cause should act so uniformly in the same way, as to make the error of the mean result nearly

equal to the difference between this and the extreme; and, therefore, it seems very unlikely that the density of the earth should differ from 5,48 by so much as $\frac{1}{14}$ of the whole.

Another objection, perhaps, may be made to these experiments, namely, that it is uncertain whether, in these small distances, the force of gravity follows exactly the same law as in greater distances. There is no reason, however, to think that any irregularity of this kind takes place, until the bodies come within the action of what is called the attraction of cohesion, and which seems to extend only to very minute distances. With a view to see whether the result could be affected by this attraction, I made the 9th, 10th, 11th, and 15th experiments, in which the balls were made to rest as close to the sides of the case as they could; but there is no difference to be depended on, between the results under that circumstance, and when the balls are placed in any other part of the case.

According to the experiments made by Dr. MASKELYNE, on the attraction of the hill Schehallien, the density of the earth is $4\frac{1}{2}$ times that of water; which differs rather more from the preceding determination than I should have expected. But I forbear entering into any consideration of which determination is most to be depended on, till I have examined more carefully how much the preceding determination is affected by irregularities whose quantity I cannot measure.

APPENDIX.

On the Attraction of the Mahogany Case on the Balls.

THE first thing is, to find the attraction of the rectangular plane $c\,k\,\beta\,b$ (fig. 8.) on the point a, placed in the line $a\,c$ perpendicular to this plane.

Let $a\,c = a$, $c\,k = b$, $c\,b = x$, and let $\dfrac{a^2}{a^2 + x^2} = w^2$, and $\dfrac{b^2}{a^2 + x^2} = v^2$, then the attraction of the line $b\,\beta$ on a, in the direction $a\,b, = \dfrac{b\,\beta}{a\,b \times a\,\beta}$; and therefore, if $c\,b$ flows, the fluxion of the attraction of the plane on the point a, in the direction $c\,b$,

$$= \frac{b\,\dot{x}}{\sqrt{a^2 + x^2} \times \sqrt{a^2 + b^2 + x^2}} \times \frac{x}{\sqrt{a^2 + x^2}} = \frac{-b\,\dot{v}}{w\sqrt{b^2 + \dfrac{a^2}{w^2}}} = \frac{-b\,\dot{v}}{\sqrt{b^2 w^2 + a^2}} = \frac{-\dot{v}}{\sqrt{1 + v^2}},$$

the variable part of the fluent of which $= -\log. v + \sqrt{1 + v^2}$, and therefore the whole attraction $= \log. \overline{\dfrac{c\,k + a\,k}{a\,c}} \times \overline{\dfrac{a\,b}{b\,\beta + a\,\beta}}$; so that the attraction of the plane, in the direction $c\,b$, is found readily by logarithms, but I know no way of finding its attraction in the direction $a\,c$, except by an infinite series.

The two most convenient series I know, are the following:

First series. Let $\dfrac{b}{a} = \pi$, and let $A =$ arc whose tang. is π, $B = A - \pi$, $C = B + \dfrac{\pi^3}{3}$, $D = C - \dfrac{\pi^5}{5}$, &c. then the attraction in the direction $a\,c = \sqrt{1 - w^2} \times A + \dfrac{B\,w^2}{2} + \dfrac{3\,C\,w^4}{2\cdot4} + \dfrac{3\cdot5\,w^6}{2\cdot4\cdot6}$, &c.

3 X 2

For the second series, let $A =$ arc whose tang. $= \frac{1}{\pi}$, $B = A - \frac{1}{\pi}$, $C = B + \frac{1}{3\pi^3}$, $D = C - \frac{1}{5\pi^5}$, $\&c.$ then the attraction $=$ arc $. 90°$

$$-\sqrt{1 + v^2} \times A - \frac{B v^2}{2} + \frac{3 C v^4}{2.4} - \frac{3.5 D v^6}{2.4.6}, \&c.$$

It must be observed, that the first series fails when π is greater than unity, and the second, when it is less; but, if b is taken equal to the least of the two lines ck and cb, there is no case in which one or the other of them may not be used conveniently.

By the help of these series, I computed the following table.

	,1962	,3714	,5145	,6248	,7071	,7808	,8575	,9285	,9815	1,
,1962	,00001									
,3714	,00039	00148								
,5145	,00074	00277	00521							
,6248	00110	00406	00778	01183						
,7071	00140	00522	01008	01525	02002					
,7808	00171	00637	01245	01896	02405	03247				
,8575	00207	00772	01522	02339	03116	03964	05057			
,9285	00244	00910	01810	02807	03778	04867	06319	08119		
,9815	00271	01019	02084	03193	04368	05639	07478	09931	12849	
1.	00284	01054	02135	03347	04560	05975	07978	10789	14632	19612

Find in this table, with the argument $\frac{ck}{ak}$ at top, and the argument $\frac{cb}{ab}$ in the left hand column, the corresponding logarithm; then add together this logarithm, the logarithm of $\frac{ck}{ak}$, and the logarithm of $\frac{cb}{ab}$; the sum is logarithm of the attraction.

To compute from hence the attraction of the case on the ball, let the box DCBA, (fig. 1.) in which the ball plays, be divided into two parts, by a vertical section, perpendicular to the length of the case, and passing through the centre of the ball; and, in fig. 9, let the parallelopiped ABDE $abde$ be one of these parts, ABDE being the abovementioned vertical section; let x be the centre of the ball, and draw the parallelogram $\beta n p m \delta x$ parallel to B$b d$D, and $xgrp$ parallel to βBbn, and bisect $\beta \delta$ in c. Now, the dimensions of the box, on the inside, are Bb=1,75; BD=3,6; Bβ=1,75; and βA=5; whence I find, that if xc and βx are taken as in the two upper lines of the following table, the attractions of the different parts are as set down below.

	,75	,5	,25
xc - -	,75	,5	,25
βx - -	1,05	1,3	1,55
Excess of attract. of Ddrg above Bbrg	,2374	,1614	,0813
———— $mdrp$ above $nbrp$	,2374	,1614	,0813
———— $mesp$ above $nasp$	,3705	,2516	,1271
Sum of these -	,8453	,5744	,2897
Excess of attract. of B$bn\beta$ above D$dm\delta$	,5007	,3271	,1606
———— A$an\beta$ above E$em\delta$	,4677	,3079	,1525
Whole attraction of the inside surface of the half box - - -	,1231	,0606	,0234

It appears, therefore, that the attraction of the box on x increases faster than in proportion to the distance xc.
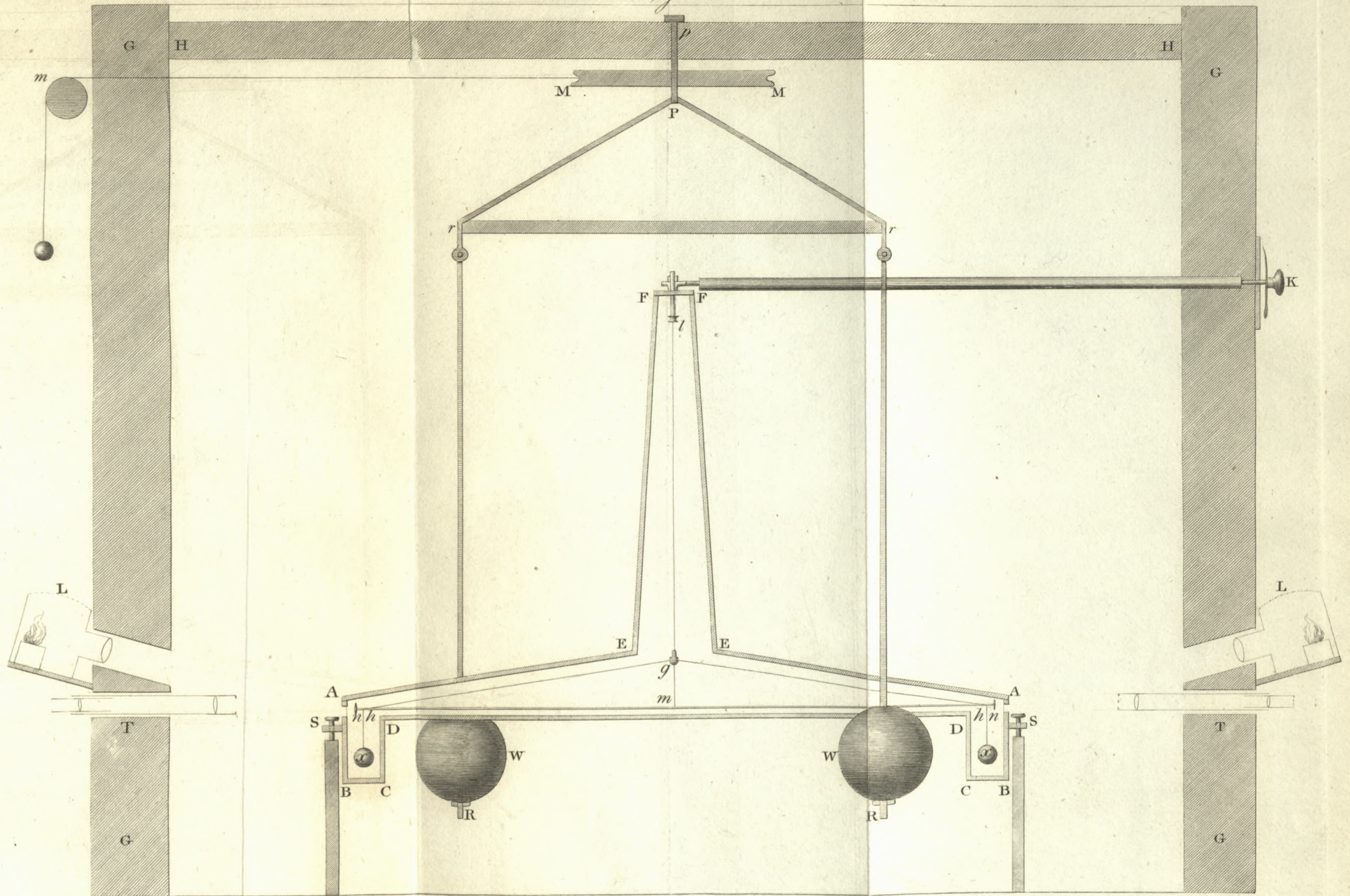
The specific gravity of the wood used in this case is ,61, and its thickness is $\frac{3}{4}$ of an inch; and therefore, if the attraction of the outside surface of the box was the same as that of the inside, the whole attraction of the box on the ball, when cx = ,75,

would be equal to $2 \times ,1231 \times ,61 \times \frac{3}{4}$ cubic inches, or ,201 spheric inches of water, placed at the distance of one inch from the centre of the ball. In reality, it can never be so great as this, as the attraction of the outside surface is rather less than that of the inside; and, moreover, the distance of x from c can never be quite so great as ,75 of an inch, as the greatest motion of the arm is only $1\frac{1}{2}$ inch.

Fig. 1.

Basire Sc.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.